# Automatic model decomposition in Hexaly

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Hexaly Optimizer is a model-and-run solver based on various heuristics and exact methods. A set-based modeling formalism was introduced to simplify the modeling of certain combinatorial problems such as routing, scheduling or packing problems. This talk will present how Hexaly uses automatic model decomposition and branch price and cut methods to compute lower bounds for several vehicle routing problems.

## 1 Modeling routing problems with set decisions

Vehicle routing problems can be easily modeled using `list` decisions in Hexaly's modeling language. A `list(n)` represents an ordered subset of integers between 0 and n (excluded) and can be used to model the sequence of customers visited by one truck. A `partition` constraint on all the lists ensures that each customer is served by exactly one truck. Finally, functional expressions can be used to model capacity constraints and the total traveled distance to model a standard CVRP. Other expressions can be used to model VRP variants, such as time windows, fix costs per truck, etc.

Set-based models are compact with a number of `list` decisions equals to the number of trucks. This model is well suited for a heuristic search but is much more difficult to solve with a mathematical programming approach to compute lower bounds.

## 2 Approach

Some set-based models with a `partition` constraint can be expressed as set partition problems with an exponential number of binary variables. Each variable models the selection of a feasible list assignment in the set-based model. A first block of set partition constraints ensures that each client is visited by exactly one selected assignment. A final constraint ensures that at most one assignment per list is selected. Solving the explicit model is not tractable in practice, but efficient techniques such as column generation and branch price and cut [1, 2] can compute a lower bound on this problem. Solutions are computed with a heuristic approach.

Hexaly automatically performs this reformulation on compatible set-based models, discarding incompatible constraints to compute lower bounds on most vehicle routing problems.

# 3 Results

The method is benchmarked on standard instances from the literature [3] for CVRP and CVRPTW. The solver is launched on all instances for 60s and we measure the gap reported when the time limit is reached. For instances with more than 200 clients the approach is not competitive with a standard arc model and subtour elimination constraints and the solver falls back to the arc model. Table 1 shows for each dataset the percentage of instances with a lower bound computed using automatic model reformulation and the average gap reported by this technique after 60s.

| Type | Instances | Compatible inst. | Average gap |
|---|---|---|---|
| CVRP | Set A | 100% | 0.11% |
| CVRP | Set B | 100% | 0.06% |
| CVRP | Set X ($n \leq 200$) | 72.73% | 3.74% |
| CVRPTW | Solomon | 59% | 0.6% |
| HVRP | DLP set | 32.29% | 10.9% |

Table 1: Average gap reported by Hexaly with automatic model reformulation in 60s

Hexaly reports near-optimal solutions and gaps after 60s on CVRP and CVRPTW instances with less than 200 customers. This approach will be extended to other practical vehicle routing constraints such as pickup and delivery, site-dependent VRP, etc.

# References


[1] Artur Alves Pessoa, Ruslan Sadykov, Eduardo Uchoa, and François Vanderbeck. A Generic Exact Solver for Vehicle Routing and Related Problems. *Mathematical programming*, 183:483–523, 2020.

[2] Jacques Desrosiers, Marco Lübbecke, Guy Desaulniers, Jean Bertrand Gauthier. Branch-and-Price 10.13140/RG.2.2.29888.14088.

[3] http://vrp.galgos.inf.puc-rio.br/index.php/en/updates